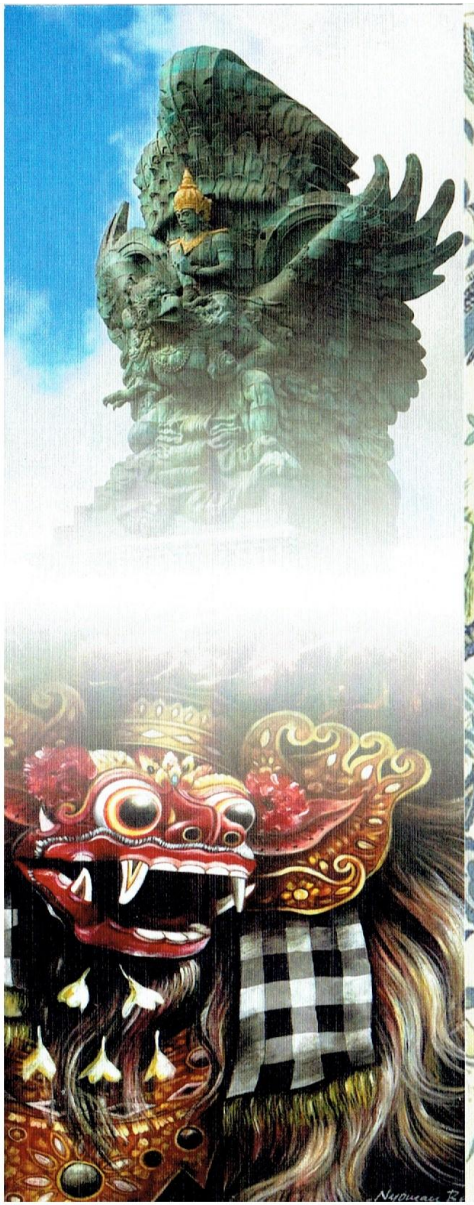

The 2nd International Conference
on Mathematics and Natural Sciences 2019

**Faculty of Mathematics and Natural Sciences**
**Universitas Pendidikan Ganesha**

IC on MNS

# Certificate of Appreciation

NO : 1384/UN48.9/DL/2019

**This is to certify that**

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has participated as a ***Presenter***

with a talk entitled

*On Magic and Antimagic Total Labelings of Graphs*

at the 2nd International Conference on Mathematics and Natural Sciences 2019

held on August 30 - 31, 2019 at Harris Sunset Road, Kuta-Bali

Denpasar, August 31, 2019

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IC on MNS International Conference on Mathematics and Natural Sciences (IConMNS)

Dr. I Gede Aris Gunadi,S.Si.,M.Kom.
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# On magic and antimagic total labelings of graphs

Anak Agung Gede Ngurah

Universitas Merdeka Malang, Indonesia

International Conference on Mathematics and Natural Sciences
(IConMNS 2019)
Kuta - Bali, August 30 - 31, 2019

# Graph Labeling

- $G$ is a finite and simple graph.

$$V(G) = \text{vertex set}; \ |V(G)| = p,$$
$$E(G) = \text{edge set}; \ |E(G)| = q.$$

- A *labeling* of a graph $G$ is a one to one mapping from some set of graph elements to a set of positive integers.
  - A *vertex labeling* $f : V(G) \to \{1, 2, 3, \ldots, p\}$.
  - An *edge labeling* $f : E(G) \to \{1, 2, 3, \ldots, q\}$.
  - A *total labeling* $f : V(G) \cup E(G) \to \{1, 2, 3, \ldots, p + q\}$.

# Weight

Let $f$ be a total labeling of $G$.

- *Vertex-weight* $w(v)$, $v \in V(G)$:
  Sum of label of $v$ and labels of its incident edges;

$$w(v) = f(v) + \sum_{u \in N(v)} f(uv).$$

- *Edge-weight* $w(e)$, $e = uv \in E(G)$:
  Sum of label of $e$ and of labels of its endpoints;

$$w(uv) = f(u) + f(uv) + f(v).$$

# Magic (Antimagic) Labeling

- *Vertex-magic (vertex-antimagic) total labelings.*
- *Edge-magic (edge-antimagic) total labelings.*

# Edge-Magic Total Labeling

- An *edge-magic total* (EMT) *labeling* of a graph $G$ with $p$ vertices and $q$ is a bijective function

$$f : V(G) \cup E(G) \to \{1, 2, 3, \cdots, p + q\}$$

  such that $f(x) + f(xy) + f(y) = k_f$ is a constant for any edge $xy$ of $G$.
- $G$ is called an EMT *graph.*
- $k_f$ is called *magic constant* of $f$.

A. Kotzig and A. Rosa, Magic valuation of finite graphs, *Canad. Math. Bull.*, **13** (4), (1970), 451 - 461.

# Super Edge-Magic Total Labeling

- An EMT labeling $f$ of $G$ is called a *super edge-magic total* (SEMT) *labeling* if $f(V(G)) = \{1, 2, 3, \cdots, p\}$.
- $G$ is called a SEMT *graph.*

H. Enomoto, A. Llado, T. Nakamigawa, and G. Ringel, Super edge-magic graphs, *SUT J. Math.*, **34** (1998), 105 - 109.

# Example

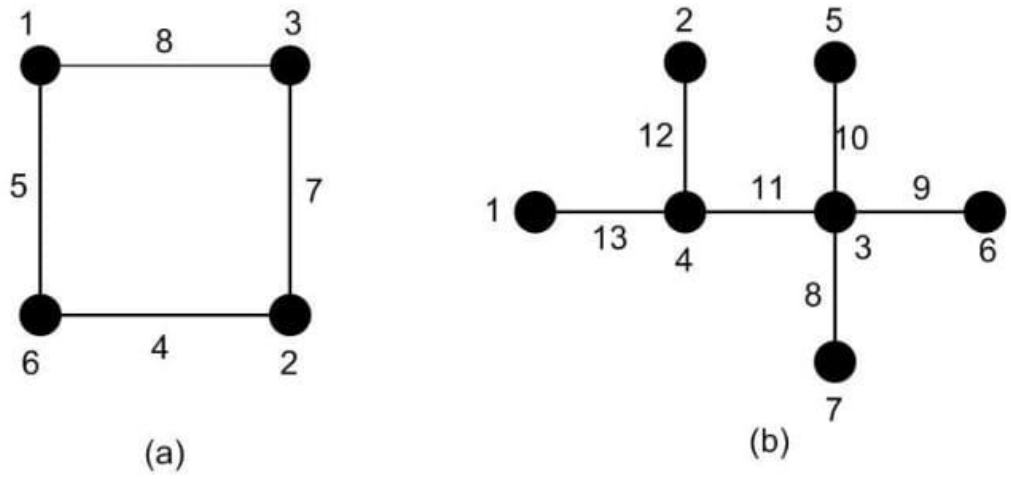


Figure: An EMT graph with $k = 12$ and a SEMT graph $k = 18$ .

# $(a, d)$-Edge-Antimagic Total Labeling

- An *$(a, d)$-edge-antimagic total* ($(a, d)$-EAT) labeling of a graph $G$ with $p$ vertices and $q$ edges is a bijective function

$$f : V(G) \cup E(G) \to \{1, 2, 3, \cdots, p + q\}$$

  such that $\{f(x) + f(xy) + f(y) : xy \in E(G)\}$ is equal to $\{a, a + d, a + 2d, \ldots, a + (q - 1)d\}$, for two integers $a > 0$ and $d \geq 0$.
- $G$ is called an $(a, d)$-EAT*graph.*

R. Simanjuntak, F. Bertault and M. Miller, Two new $(a, d)$-antimagic graph labelings , *Proc. of Eleventh Australian Workshop on Combinatorial Algorithm,* (2000), 179 - 184.

# Super $(a, d)$-Edge-Antimagic Total Labeling

- An $(a, d)$-EAT labeling $f$ is called a *super* $(a, d)$-EAT *labeling* if $f(V(G)) = \{1, 2, 3, \ldots, p\}$.
- $G$ is a *super* $(a, d)$-EAT *graph.*

Note: when $d = 0$, a (super) $(a, 0)$-EATlabeling is in fact a (super) EMT labeling.

R. Simanjuntak, F. Bertault and M. Miller, Two new $(a, d)$-antimagic graph labelings , *Proc. of Eleventh Australian Workshop on Combinatorial Algorithm,* (2000), 179 - 184.

# Example

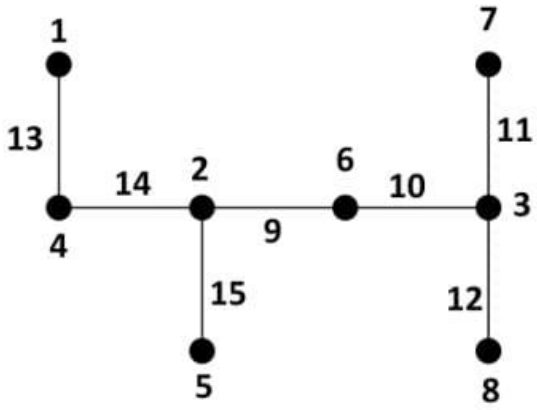


Figure: A super (17, 1)-EAT graph.

## Necessary and Sufficient conditions

- A graph $G$ is super EMT if and only if there is a bijective function $f : V(G) \longrightarrow \{1, 2, 3, \ldots, p\}$ such that the set $S = \{f(x) + f(y) | xy \in E(G)\}$ is a set of $q$ consecutive integers. In this case, $f$ can be extended to a super EMT labeling of $G$ with magic constant $p + q + min(S)$.

R. Figueroa-Centeno, R. Ichishima, and F. A. Muntaner-Batle, The place of super edge-magic labelings among other classes of labelings, *Discrete Math.*, **231** (2001), 153 – 168.

# Example

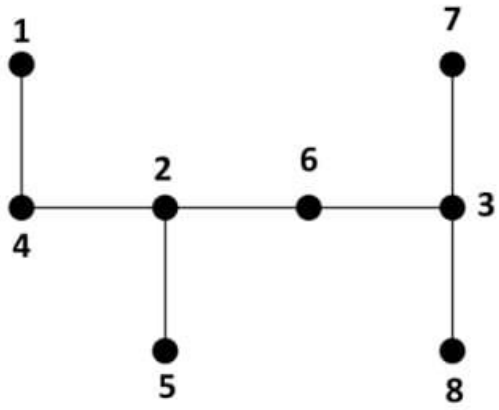


Figure: A vertex labeling of a tree with $S = \{5, 6, 7, 8, 9, 10, 11\}$.

# Relationships of a Super EMT Labeling with Other Labelings

- If a graph $G$ that is a tree or where $q \geq p$ is super EMT, then $G$ is sequential, harmonious, cordial. [1].
- Suppose that $G$ is a super EMT bipartite graph with partite sets $V_1$ and $V_2$ and let $f$ be a super EMT labeling of $G$ such that $f(V_1) = \{1, 2, 3, \ldots, |V_1|\}$, then $G$ has an $\alpha$-labeling. [1].


[1]. R. Figueroa-Centeno, R. Ichishima, and F. A. Muntaner-Batle, The place of super edge-magic labelings among other classes of labelings, *Discrete Math.*, **231** (2001), 153 – 168.

[2]. J. Gallian, A dynamic survey of graph labeling, *Electron. J. Combin.*, **DS6** (2018) http://www.combinatorics.org.

## Relationships of a Super EMT Labeling with Other Labelings

- Let $G$ be a graph which admits total labeling and whose edge labels an arithmetic progression with difference $d$. Then the following are equivalent.
  (i). $G$ has an EMT labeling with magic constant $k$.
  (ii). $G$ has a $(k-(q-1)d, 2d)$-EAT labeling.

As a consequence of this result:

- If $G$ has a super EMT labeling with magic constant $k$, then $G$ has a super $(k-q+1, 2)$-EAT labeling.

M. Baca, Y. Lin, M. Miller, and R. Simanjuntak, New contructions of magic and antimagic graph labelings, *Util. Math.*, **60**, (2001), 229 - 239.

## Results

(AAGN, 2019++) Let $G$ be a super EMT graph with magic constant $k$.

(1). If $q$ is odd, then $G$ is a super $(\alpha, 1)$-EAT graph, where $\alpha = k - \frac{1}{2}(q-1)$.

(2). If $q = p - 1$ or $q = p$, then $G$ is an $(\alpha, 4)$-EAT graph, where $\alpha = 2k - 2(p+q)$, such that all vertices receive the odd labels.

(3). If $q = p - 1$ or $q = p$ and $q$ is odd, then $G$ is an $(\alpha, 2)$-EAT graph, where $\alpha = 2k - (2p + q + 1)$, such that all vertices receive the odd labels.

(4). If $q = p$, then $G$ is an $(\alpha, 4)$-EAT graph, where $\alpha = 2k - 2(p+q) + 1$, such that all vertices receive the even labels.

# Example

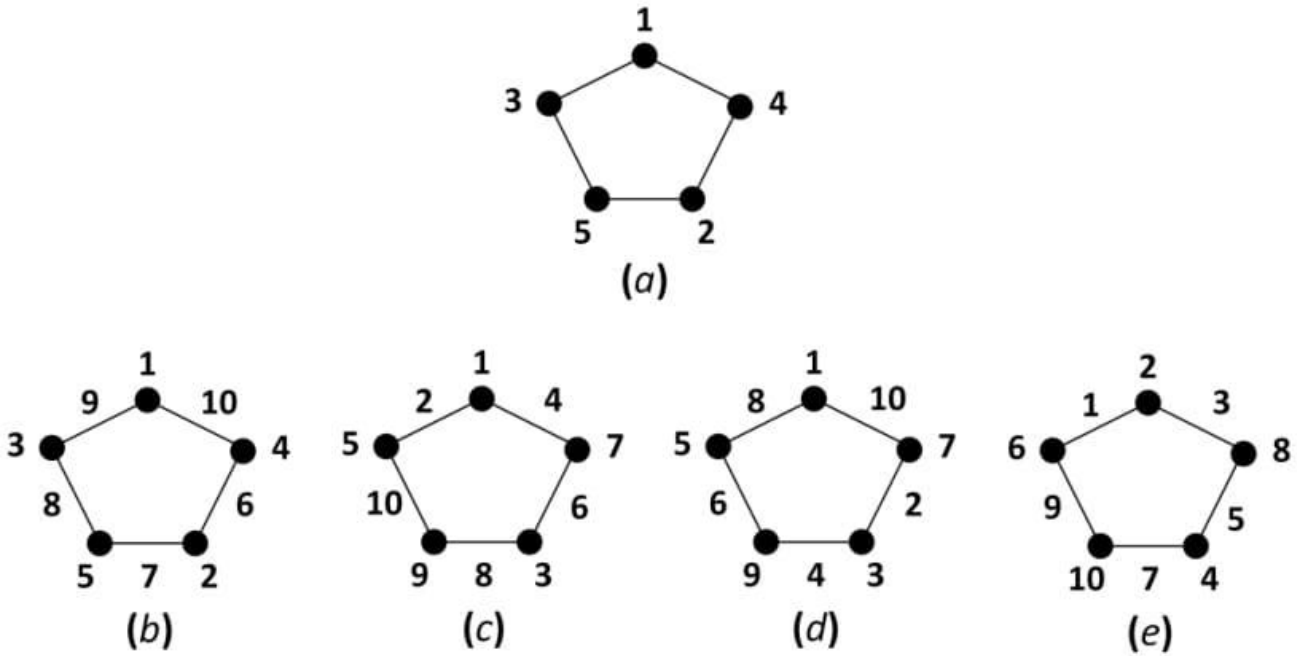


Figure: (a). A super EMT labeling of $C_5$. (b). A super (12, 1)-EAT labeling of $C_5$. (c). An (8, 4)-EAT labeling of $C_5$. (d). An (12, 2)-EAT labeling of $C_5$. (e). An (9, 4)-EAT labeling of $C_5$.

## Results; 2-regular Graphs

Holden et al. proved that,

- $C_5 \cup (2t)C_3$ is SEMT for each integer $t \geq 3$.
- $C_4 \cup (2t-1)C_3$ is SEMT for each integer $t \geq 3$.
- $C_7 \cup (2t)C_3$ is SEMT for each integer $t \geq 1$.

- **Conjectured** : All 2-regular graphs of odd order are SEMT, excluding $C_3 \cup C_4$, $3C_3 \cup C_4$ and $2C_3 \cup C_5$.

J. Holden, D. McQuillan, and J. M. McQuillan, A conjecture on strong magic labelings of 2-regular graphs, *Discrete Math.*, **312**, (2009), 4130–4136.

## Results; 2-regular Graphs

(AAGN, 2019++) All the following graphs are super $(\alpha', 1)$-EAT graphs, $(\alpha'', 2)$-EAT graphs, and $(\alpha''', 4)$-EAT graphs, for some integers $\alpha', \alpha''$ and $\alpha'''$.

- $C_5 \cup (2t)C_3$ is SEMT for each integer $t \geq 3$.
- $C_4 \cup (2t-1)C_3$ is SEMT for each integer $t \geq 3$.
- $C_7 \cup (2t)C_3$ is SEMT for each integer $t \geq 1$.

# Results: 2-regular Graphs

Figueroa-Centeno et al. proved that:

- $C_3 \cup C_n$ is SEMT iff $n \geq 6$ and $n$ is even.
- $C_4 \cup C_n$ is SEMT iff $n \geq 5$ and $n$ is odd.
- $C_5 \cup C_n$ is SEMT iff $n \geq 4$ and $n$ is even.
- $C_n \cup C_m$ is SEMT if $n$ is even and $m \geq \frac{n}{2} + 1$ is odd.

R. M. Figueroa-Centeno, R. Ichishima, and F. A. Muntaner-Batle, A magical approach to some labeling conjectures, *Discuss. Math. Graph Theory*, **31**, (2011), 79–113.

## Results: 2-regular Graphs

(AAGN, 2019++) All the following graphs are super $(\alpha', 1)$-EAT graphs, $(\alpha'', 2)$-EAT graphs, and $(\alpha''', 4)$-EAT graphs, for some integers $\alpha', \alpha''$ and $\alpha'''$.

- $C_3 \cup C_n$ is SEMT iff $n \geq 6$ and $n$ is even.
- $C_4 \cup C_n$ is SEMT iff $n \geq 5$ and $n$ is odd.
- $C_5 \cup C_n$ is SEMT iff $n \geq 4$ and $n$ is even.
- $C_n \cup C_m$ is SEMT if $n$ is even and $m \geq \frac{n}{2} + 1$ is odd.

## Results: 2-regular graphs

(AAGN, 2019++) If $m, t \geq 3$, $n \geq 4$, and $l \geq 6$ are positive integers such that $m$ and $t$ are odd and $l \equiv 2 \pmod 4$, then all the following graphs are super EMT.

a). $m[C_{nt} \cup C_t]$ for $n \equiv 0 \ (mod\ 2)$.

b). $m[C_{nt} \cup 2C_t]$ for $n \equiv 1 \ (mod\ 2)$.

c). $m[C_{4t} \cup C_{nt} \cup C_t]$ for $n \equiv 0 \ (mod\ 4)$.

d). $m[C_{4t} \cup C_{nt} \cup C_t]$ for $n \in \{6, 10, 14, 18, 22, 26\}$.

e). $lC_{4t} \cup m[C_{nt} \cup C_t]$ for $n \equiv 8, 12 \ (mod\ 16)$.

## Results: 2-regular graphs

(AAGN, 2019++) If $m, t \geq 3$, $n \geq 4$, and $l \geq 6$ are positive integers such that $m$ and $t$ are odd and $l \equiv 2 \pmod 4$, then all the following graphs are super $(\alpha', 1)$-EAT graphs, $(\alpha'', 2)$-EAT graphs, and $(\alpha''', 4)$-EAT graphs, for some integers $\alpha', \alpha''$ and $\alpha'''$.

a). $m[C_{nt} \cup C_t]$ for $n \equiv 0 \ (mod\ 2)$.

b). $m[C_{nt} \cup 2C_t]$ for $n \equiv 1 \ (mod\ 2)$.

c). $m[C_{4t} \cup C_{nt} \cup C_t]$ for $n \equiv 0 \ (mod\ 4)$.

d). $m[C_{4t} \cup C_{nt} \cup C_t]$ for $n \in \{6, 10, 14, 18, 22, 26\}$.

e). $lC_{4t} \cup m[C_{nt} \cup C_t]$ for $n \equiv 8, 12 \ (mod\ 16)$.

## Results: 2-regular Graphs

- Let $m$ be an odd integer. If $G \cong \bigcup_{i=1}^{k} C_{n_i}$ is super EMT, then $H \cong \bigcup_{i=1}^{k} (m, n_i) C_{[m,n_i]}$ is super EMT, where
  - $(a, b)$ is the *greatest common divisor* of $a$ and $b$,
  - $[a, b]$ is the *least common multiple* of $a$ and $b$.

R. Ichishima, F. A. Muntaner-Batle, and A. Oshima, Enlarging the classes of super edge-magic 2-regular graphs , *AKCE Int. J. Graphs Comb.*, **10** (2), (2013), 129–146.

# Results: 2-regular Graphs

(AAGN, 2019++) Let $m$ be an odd integer. If $G \cong \bigcup_{i=1}^{k} C_{n_i}$ is super EMT, then $H \cong \bigcup_{i=1}^{k} (m, n_i) C_{[m,n_i]}$ is a super $(\alpha', 1)$-EAT graph, an $(\alpha'', 2)$-EAT graph, and an $(\alpha''', 4)$-EAT graph, for some integers $\alpha', \alpha''$ and $\alpha'''$.

, where

- $(a, b)$ is the *greatest common divisor* of $a$ and $b$,
- $[a, b]$ is the *least common multiple* of $a$ and $b$.

## Results: Trees

Let $K_{n_1}, K_{n_2}, \ldots, K_{n_t}$, be a family of disjoint stars. Let $v_i$ be a pendant vertex of $G_i$, $1 \leq i \leq t$. The tree which contains all the $t$ stars and a path joining $v_1, v_2, \ldots, v_t$ is called a *firecracker* and it is denoted by $FC(n_1, n_2, \ldots, n_t)$.

- $FC(n_1, n_2, \ldots, n_t)$ is a super EMT graph, if $n_1 = n_2 = \ldots = n_t$. [1]
- $FC(n_1, n_2, \ldots, n_t)$ is a super EMT graph, if $n_1 \leq n_2 \leq \ldots \leq n_t$. [2]


[1]. V. Swaminathana and P. Jeyanthi, Super edge-magic strength of fire crackers, banana trees and unicyclic graphs, *Discrete Math.*, **306** (14) (2018), 1624 - 1636.

[2]. E. T. Baskoro, R. Simanjuntak, S. Uttunggadewa, and AAGN On super edge-magic strength and deficiency of graphs, *LNCS*, **4535** (2008), 144 - 154.

# Results: Trees

(AAGN, 2019++)

- If $n_2 \leq n_3 \ldots \leq n_t$ and $n_i = n_{2t+1-i}$, $2 \leq i \leq t$, then $\mathrm{FC}(n_1, n_2, \ldots, n_{2t})$ is a super EMT graph.
- If $n_2 \leq n_3 \ldots \leq n_t$ and $n_i = n_{2t+1-i}$, $2 \leq i \leq t$, then $\mathrm{FC}(n_1, n_2, \ldots, n_{2t})$ is a super $(\alpha', 1)$-EAT graph, an $(\alpha'', 2)$-EAT graph, and an $(\alpha''', 4)$-EAT graph, for some integers $\alpha', \alpha''$ and $\alpha'''$.

# Example

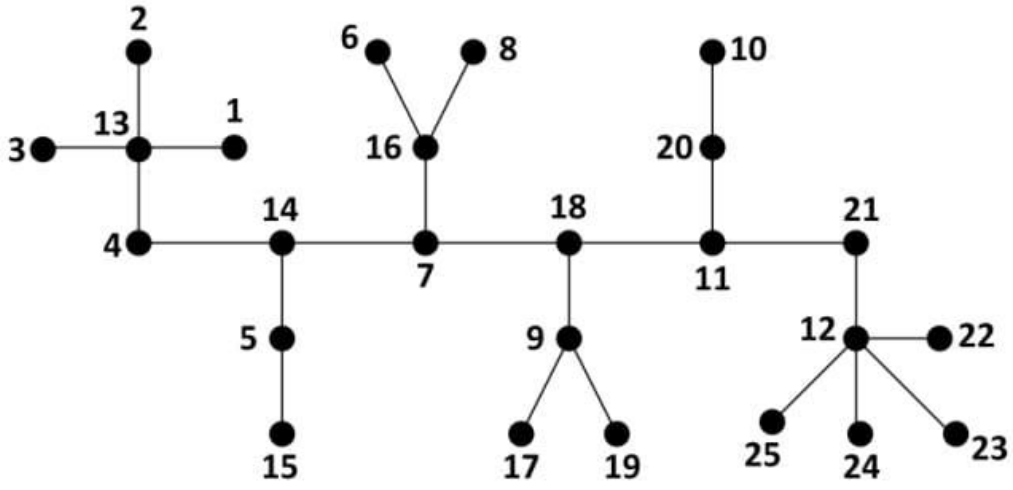


Figure: A vertex labeling of $FC(4, 2, 3, 3, 2, 5)$.

# Results: Trees

The *corona product* of $G$ and $nK_1$, $G \odot nK_1$, is a graph obtained by attaching $n$ isolated vertices to each vertex of $G$.

T(n, m) , $n \geq 2$ and $m \geq 1$, is a tree constructed from a caterpillar $P_n \odot 2K_1$ by inserting $m$ vertices to every pendant of $P_n \odot 2K_1$.

(AAGN, 2019++)

- For every $n \geq 2$ and $m \geq 1$, T(n, m) is a super EMT graph.
- For every $n \geq 2$ and $m \geq 1$, T(n, m) is a super $(\alpha', 1)$-EAT graph, an $(\alpha'', 2)$-EAT graph, and an $(\alpha''', 4)$-EAT graph, for some integers $\alpha', \alpha''$ and $\alpha'''$.

# Example

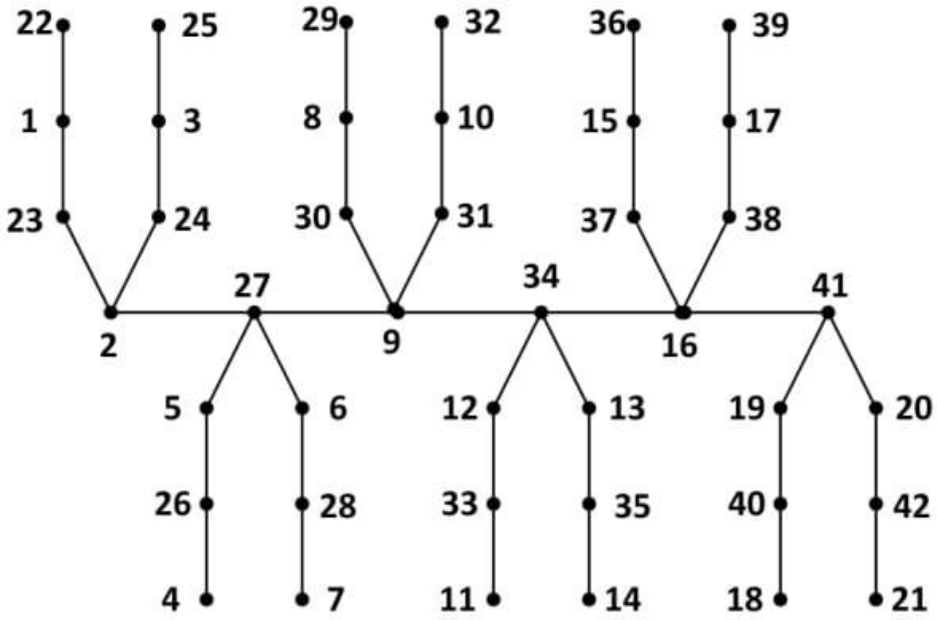


Figure: A vertex labeling of $T(6, 2)$.

## Results: Unicyclic graphs

- [1]. Let $m, n$ and $t$ be positive integers such that $m, n \geq 3$ are odd. Then $m(C_n \odot tK_1)$ is a super EMT graph.
- (AAGN, 2019++) Let $m, n$ and $t$ be positive integers such that $m, n \geq 3$ are odd. Then $m(C_n \odot tK_1)$ is a super $(\alpha', 1)$-EAT graph, an $(\alpha'', 2)$-EAT graph, and an $(\alpha''', 4)$-EAT graph, for some integers $\alpha', \alpha''$ and $\alpha'''$.

[1]. J. Gallian, A dynamic survey of graph labeling, *Electron. J. Combin.*, **DS6** (2018) http://www.combinatorics.org.

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- M. Baća and M. Miller, Super Edge-Antimagic Graphs, Brown - Walker Press, Boca Raton, 2008.

THANK YOU FOR YOUR ATTENTION